

LIGHT AXIAL-VECTOR TETRAQUARK STATE CANDIDATE: $a_1(1420)$ Zhi-Gang Wang¹

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Abstract

In this article, we study the axial-vector tetraquark state and two-quark-tetraquark mixed state consist of light quarks using the QCD sum rules. The present predictions disfavor assigning the $a_1(1420)$ as the axial-vector tetraquark state with $J^{PC} = 1^{++}$, while support assigning the $a_1(1420)$ as the axial-vector two-quark-tetraquark mixed state.

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1 Introduction

Recently the COMPASS collaboration observed the $a_1(1420)$ in the $f_0(980)\pi$ final state in the reaction $\pi^- + p \rightarrow \pi^- \pi^- \pi^+ + p_{\text{recoil}}$, the measured mass and width are 1420 MeV and 140 MeV, respectively [1]. Although there are controversies about the quark structures of the $f_0(980)$, it contains some $s\bar{s}$ constituents, irrespective of the two-quark state, molecular state or tetraquark state assignments [2]. The $a_1(1420)$ is excellent candidate of the axial-vector tetraquark state with the symmetric spin structure $[su]_{S=1}[\bar{s}\bar{d}]_{S=0} + [su]_{S=0}[\bar{s}\bar{d}]_{S=1}$. On the other hand, the axial-vector meson $f_1(1420)$ with $J^{PC} = 1^{++}$ has the mass and width (1426.4 ± 0.9) MeV and (54.9 ± 2.6) MeV, respectively [3]. The width of the $f_1(1420)$ is much smaller than that of the $a_1(1420)$, it is unlikely that the $a_1(1420)$ and $f_1(1420)$ are the same particle.

In this article, we take the $a_1(1420)$ as the axial-vector tetraquark state, study its mass and pole residues using the QCD sum rules, and make possible assignment of the $a_1(1420)$ in the tetraquark scenario. The QCD sum rules is a powerful theoretical tool is studying the ground state hadrons and has given many successful descriptions of the masses, decay constants, form-factors, coupling constants, etc [4, 5].

The article is organized as: we derive the QCD sum rules for the axial-vector tetraquark states in section 2; in section 3 we present numerical results and discussions; while the last section is reserved for our conclusion.

2 QCD sum rules for the axial-vector tetraquark states

We write down the two-point correlation functions $\Pi_{\mu\nu}(p)$ in the QCD sum rules firstly,

$$\Pi_{\mu\nu}(p) = i \int d^4x e^{ip \cdot x} \langle 0 | T \{ J_\mu(x) J_\nu^\dagger(0) \} | 0 \rangle, \quad (1)$$

$$J_\mu^1(x) = \frac{\epsilon^{ijk} \epsilon^{imn}}{\sqrt{2}} \{ u^j(x) C \gamma_5 s^k(x) \bar{d}^m(x) \gamma_\mu C \bar{s}^n(x) + u^j(x) C \gamma_\mu s^k(x) \bar{d}^m(x) \gamma_5 C \bar{s}^n(x) \},$$

$$J_\mu^2(x) = \frac{\epsilon^{ijk} \epsilon^{imn}}{\sqrt{2}} \{ u^j(x) C \gamma_5 s^k(x) \bar{d}^m(x) \gamma_\mu C \bar{s}^n(x) - u^j(x) C \gamma_\mu s^k(x) \bar{d}^m(x) \gamma_5 C \bar{s}^n(x) \},$$

$$J_\mu^3(x) = \cos\theta J_\mu^1(x) + \sin\theta \frac{2\sqrt{2}}{3} \langle \bar{s}s \rangle \bar{d}(x) \gamma_5 \gamma_\mu u(x), \quad (2)$$

where the currents $J_\mu(x) = J_\mu^1(x), J_\mu^2(x), J_\mu^3(x)$, the i, j, k, m, n are color indexes, the C is the charge conjunction matrix. The factor $\frac{2\sqrt{2}}{3} \langle \bar{s}s \rangle$ is added to normalize the current $J_\mu^3(x)$, the θ is the mixing angle. The currents $J_\mu^1(x)$ and $J_\mu^3(x)$ have the quantum numbers $J^{PC} = 1^{++}$, we choose

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them to interpolate the tetraquark state and two-quark-tetraquark mixed state, respectively. The current $J_\mu^2(x)$ has the quantum numbers $J^{PC} = 1^{+-}$, we choose it to study the negative charge conjugation partner of the axial-vector tetraquark state as a byproduct.

At the hadronic representation, the ground states contributions from the axial-vector mesons can be written as

$$\Pi_{\mu\nu}(p) = \frac{\lambda^2}{M^2 - p^2} \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} \right) + \cdots, \quad (3)$$

where the pole residues λ are defined by

$$\langle 0 | J_\mu(0) | A(p) \rangle = \lambda \varepsilon_\mu, \quad (4)$$

the ε_μ are the polarization vectors of the axial-vector mesons A .

We calculate the contributions of the vacuum condensates up to dimension-10 in the operator product expansion, assume vacuum saturation to factorize the higher dimension vacuum condensates to lower dimension vacuum condensates, and obtain the spectral densities at the level of quark-gluon degree's of freedom through dispersion relation. Then we take the quark-hadron duality below the continuum thresholds s_0 and perform Borel transform with respect to the variable $-p^2$ to obtain three QCD sum rules,

$$\lambda^2 e^{-\frac{M^2}{T^2}} = \int_0^{s_0} dt \rho_i(t) e^{-\frac{t}{T^2}}, \quad (5)$$

where the $\rho_i(t)$ with $i = 1, 2, 3$ are the QCD spectral densities corresponding to the interpolating currents $J_\mu^1(x)$, $J_\mu^2(x)$, $J_\mu^3(x)$, respectively,

$$\begin{aligned} \rho_1(t) = & \frac{t^4}{73728\pi^6} - \frac{7m_s \langle \bar{q}q \rangle t^2}{768\pi^4} + \frac{m_s \langle \bar{s}s \rangle t^2}{256\pi^4} + \frac{5m_s \langle \bar{q}g_s \sigma Gq \rangle t}{384\pi^4} - \frac{m_s \langle \bar{s}g_s \sigma Gs \rangle t}{288\pi^4} + \frac{5\langle \bar{q}q \rangle \langle \bar{s}s \rangle t}{72\pi^2} \\ & - \frac{\langle \bar{q}q \rangle \langle \bar{s}g_s \sigma Gs \rangle + \langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle}{32\pi^2} + \frac{m_s \langle \bar{q}q \rangle \langle \bar{s}s \rangle^2}{18} \delta(t) - \frac{2m_s \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle}{9} \delta(t) \\ & + \frac{\langle \bar{q}g_s \sigma Gq \rangle \langle \bar{s}g_s \sigma Gs \rangle}{192\pi^2} \delta(t) + \frac{t^2}{9216\pi^4} \langle \frac{\alpha_s GG}{\pi} \rangle + \frac{\langle \bar{q}q \rangle \langle \bar{s}s \rangle}{144} \langle \frac{\alpha_s GG}{\pi} \rangle \delta(t) \\ & - \frac{[\langle \bar{q}q \rangle - \langle \bar{s}s \rangle]^2}{1296} \langle \frac{\alpha_s GG}{\pi} \rangle \delta(t) + \frac{5m_s [\langle \bar{q}g_s \sigma Gq \rangle - \langle \bar{s}g_s \sigma Gs \rangle] t}{3456\pi^4} \\ & + \frac{[\langle \bar{q}q \rangle - \langle \bar{s}s \rangle] [\langle \bar{q}g_s \sigma Gq \rangle - \langle \bar{s}g_s \sigma Gs \rangle]}{288\pi^2} - \frac{[\langle \bar{q}g_s \sigma Gq \rangle - \langle \bar{s}g_s \sigma Gs \rangle]^2}{1728\pi^2} \delta(t), \end{aligned} \quad (6)$$

$$\begin{aligned} \rho_2(t) = & \frac{t^4}{73728\pi^6} - \frac{7m_s \langle \bar{q}q \rangle t^2}{768\pi^4} + \frac{m_s \langle \bar{s}s \rangle t^2}{256\pi^4} + \frac{5m_s \langle \bar{q}g_s \sigma Gq \rangle t}{384\pi^4} - \frac{m_s \langle \bar{s}g_s \sigma Gs \rangle t}{288\pi^4} + \frac{5\langle \bar{q}q \rangle \langle \bar{s}s \rangle t}{72\pi^2} \\ & - \frac{\langle \bar{q}q \rangle \langle \bar{s}g_s \sigma Gs \rangle + \langle \bar{s}s \rangle \langle \bar{q}g_s \sigma Gq \rangle}{32\pi^2} + \frac{m_s \langle \bar{q}q \rangle \langle \bar{s}s \rangle^2}{18} \delta(t) - \frac{2m_s \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle}{9} \delta(t) \\ & + \frac{\langle \bar{q}g_s \sigma Gq \rangle \langle \bar{s}g_s \sigma Gs \rangle}{192\pi^2} \delta(t) + \frac{t^2}{9216\pi^4} \langle \frac{\alpha_s GG}{\pi} \rangle + \frac{\langle \bar{q}q \rangle \langle \bar{s}s \rangle}{144} \langle \frac{\alpha_s GG}{\pi} \rangle \delta(t) \\ & + \frac{[\langle \bar{q}q \rangle - \langle \bar{s}s \rangle]^2}{1296} \langle \frac{\alpha_s GG}{\pi} \rangle \delta(t) - \frac{5m_s [\langle \bar{q}g_s \sigma Gq \rangle - \langle \bar{s}g_s \sigma Gs \rangle] t}{3456\pi^4} \\ & - \frac{[\langle \bar{q}q \rangle - \langle \bar{s}s \rangle] [\langle \bar{q}g_s \sigma Gq \rangle - \langle \bar{s}g_s \sigma Gs \rangle]}{288\pi^2} + \frac{[\langle \bar{q}g_s \sigma Gq \rangle - \langle \bar{s}g_s \sigma Gs \rangle]^2}{1728\pi^2} \delta(t), \end{aligned} \quad (7)$$

$$\rho_3(t) = \cos^2 \theta \rho_1(t) + \sin^2 \theta \frac{8\langle \bar{s}s \rangle^2}{9} \left\{ \frac{t^2}{4\pi^2} - \frac{1}{12} \langle \frac{\alpha_s GG}{\pi} \rangle \delta(t) \right\}. \quad (8)$$

J^{PC}	$T^2(\text{GeV}^2)$	$\sqrt{s_0}(\text{GeV})$	pole	$M(\text{GeV})$	$\lambda(\text{GeV}^5)$
1^{++}	$0.8 - 1.1$	2.3 ± 0.1	$(66 - 92)\%$	$1.83^{+0.15}_{-0.10}$	$4.29^{+1.34}_{-0.60} \times 10^{-3}$
1^{+-}	$0.8 - 1.1$	2.3 ± 0.1	$(66 - 92)\%$	$1.82^{+0.15}_{-0.09}$	$4.27^{+1.24}_{-0.60} \times 10^{-3}$
$1^{++}(\theta = 25^\circ)$	$0.5 - 0.7$	1.9 ± 0.1	$(81 - 95)\%$	$1.42^{+0.15}_{-0.10}$	$2.35^{+0.71}_{-0.34} \times 10^{-3}$

Table 1: The Borel parameters, continuum threshold parameters, pole contributions, masses and pole residues of the axial-vector mesons.

The contribution of the three gluon condensate $\langle g_s^3 GGG \rangle$ is of the order $\mathcal{O}(\sqrt{\alpha_s}^3)$ and numerically very small, so it is neglected in this article. The differences between the spectral densities $\rho_1(t)$ and $\rho_2(t)$ are proportional to $[\langle \bar{q}g_s\sigma Gq \rangle - \langle \bar{s}g_s\sigma Gs \rangle]$, $[\langle \bar{q}q \rangle - \langle \bar{s}s \rangle]^2$, $[\langle \bar{q}q \rangle - \langle \bar{s}s \rangle][\langle \bar{q}g_s\sigma Gq \rangle - \langle \bar{s}g_s\sigma Gs \rangle]$, $[\langle \bar{q}g_s\sigma Gq \rangle - \langle \bar{s}g_s\sigma Gs \rangle]^2$, which are of minor importance and lead to almost degenerate masses for the charge conjunction partners.

We can differentiate Eq.(5) with respect to $\frac{1}{T^2}$, then eliminate the pole residues λ so as to obtain the QCD sum rules for the masses of the axial-vector mesons,

$$M^2 = \frac{\int_0^{s_0} dt \frac{d}{d(-1/T^2)} \rho_i(t) e^{-\frac{t}{T^2}}}{\int_0^{s_0} dt \rho_i(t) e^{-\frac{t}{T^2}}} . \quad (9)$$

3 Numerical results and discussions

The basic input parameters at the operator product expansion side are taken as $m_s = 0.118 \text{ GeV}$, $\langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{ GeV})^3$, $\langle \bar{s}s \rangle = (0.8 \pm 0.1) \langle \bar{q}q \rangle$, $\langle \bar{q}g_s\sigma Gq \rangle = m_0^2 \langle \bar{q}q \rangle$, $\langle \bar{s}g_s\sigma Gs \rangle = m_0^2 \langle \bar{s}s \rangle$, $m_0^2 = (0.8 \pm 0.1) \text{ GeV}^2$, $\langle \frac{\alpha_s GG}{\pi} \rangle = (0.33 \text{ GeV})^4$ at the energy scale $\mu = 1 \text{ GeV}$ [3, 4, 5, 7, 8]. We usually take the following two criteria of the QCD sum rules:

- Pole dominance at the phenomenological side;
- Convergence of the operator product expansion [6],

to choose the Borel parameters and continuum threshold parameters. It is difficult to satisfy the two criteria as the conventional two-quark mesons for the light tetraquark states [6], we have to release either of the two criteria. In other words, we can retain pole dominance by choosing small Borel parameters or retain convergence of the operator product expansion by choosing large Borel parameters.

In calculations, we observe that there appear Borel platforms (or minimum) at a special value below $T^2 = 1.0 \text{ GeV}^2$, then the hadronic masses increase monotonously with increase of the Borel parameters, although the curves of the line-shapes are not steep in some cases. On the other hand, the pole dominance is well satisfied at the region $T^2 \leq 1 \text{ GeV}^2$.

If we choose the Borel parameters at the region $T^2 \leq 1 \text{ GeV}^2$, the dominant contributions come from the $\langle \bar{q}q \rangle \langle \bar{s}s \rangle$ and $\langle \bar{q}q \rangle \langle \bar{s}g_s\sigma Gs \rangle + \langle \bar{s}s \rangle \langle \bar{q}g_s\sigma Gq \rangle$ terms rather than the perturbative terms, the vacuum condensates ($\langle \bar{q}q \rangle \langle \bar{s}s \rangle \langle \frac{\alpha_s GG}{\pi} \rangle$, $\langle \bar{q}g_s\sigma Gq \rangle \langle \bar{s}g_s\sigma Gs \rangle$, etc) of dimension 10 are of minor importance.

In this article, we retain the criterium of pole dominance and modify the criterium of convergence of the operator product expansion to be the contributions of the vacuum condensates of dimension 10 are less (or much less) than 15% of that of the $\langle \bar{q}q \rangle \langle \bar{s}s \rangle$. Then the operator product expansion is still convergent, but the convergent behavior is slower than that of the conventional two-quark mesons.

The resulting Borel parameters, continuum threshold parameters and the pole contributions are shown explicitly in Table 1. We take into account all uncertainties of the input parameters, and obtain the masses and pole residues of the axial-vector mesons, which are shown Table 1 and Fig.1.

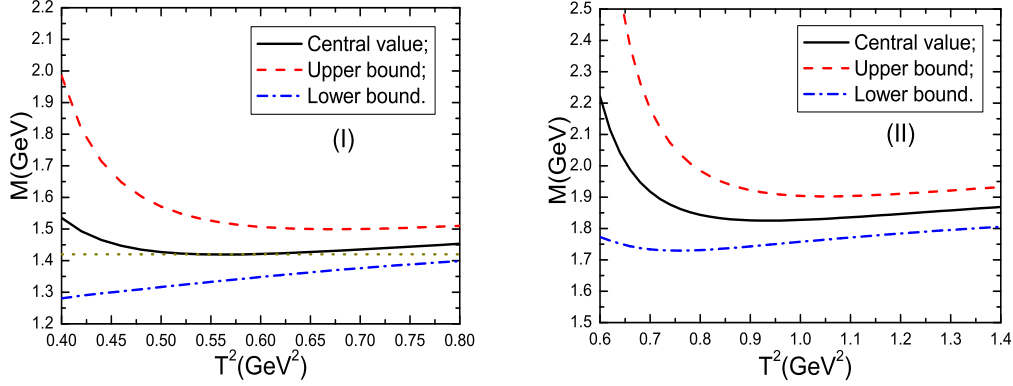


Figure 1: The masses of the axial-vector mesons with variations of the Borel parameters T^2 , where the horizontal line denotes the experimental value, the (I) and (II) denote the two-quark-tetraquark mixed state and tetraquark state with $J^{PC} = 1^{++}$, respectively.

From Table 1, we can see that the tetraquark states with the $J^{PC} = 1^{++}$ and 1^{+-} have almost degenerate masses, the values $1.83^{+0.15}_{-0.10}$ GeV and $1.82^{+0.15}_{-0.09}$ GeV are above the experimental value $M_{a_1(1420)} = 1.42$ GeV. The numerical results disfavor assigning the $a_1(1420)$ as the axial-vector tetraquark state. The mass of the axial-vector meson $a_1(1260)$ is $M_{a_1(1260)} = (1230 \pm 40)$ MeV from the Particle Data Group [3]. If we take the $a_1(1420)$ as the mixed state of the $a_1(1260)$ and tetraquark state $[su]_{S=1}[\bar{s}\bar{d}]_{S=0} + [su]_{S=0}[\bar{s}\bar{d}]_{S=1}$, the component $a_1(1260)$ can reduce the mass to the experimental value. In calculations, we observe that the mixing angle $\theta = 25^\circ$ lead to the value $1.42^{+0.15}_{-0.10}$ GeV, which reproduces the experimental data. The $a_1(1420)$ is produced through its $a_1(1260)$ component, then decays through its tetraquark component, $\pi^- + p \rightarrow a_1(1420) + p_{\text{recoil}} \rightarrow f_0(980) + \pi^- + p_{\text{recoil}} \rightarrow \pi^- \pi^- \pi^+ + p_{\text{recoil}}$.

4 Conclusion

In this article, we study the axial-vector tetraquark state and two-quark-tetraquark mixed state consist of light quarks using the QCD sum rules. The present predictions disfavor assigning the $a_1(1420)$ as the axial-vector tetraquark state with $J^{PC} = 1^{++}$, while support assigning the $a_1(1420)$ as the axial-vector two-quark-tetraquark mixed state. Furthermore, we obtain the mass of the axial-vector tetraquark state with $J^{PC} = 1^{+-}$ as a byproduct, which can be confronted with the experimental data in the future.

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References

- [1] S. Paul, arXiv:1312.3678.
- [2] C. Amsler and N. A. Tornqvist, Phys. Rept. **389** (2004) 61.

- [3] J. Beringer et al, Phys. Rev. **D86** (2012) 010001.
- [4] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. **B147** (1979) 385.
- [5] L. J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rept. **127** (1985) 1.
- [6] Z. G. Wang, Nucl. Phys. **A791** (2007) 106.
- [7] B. L. Ioffe, Prog. Part. Nucl. Phys. **56** (2006) 232.
- [8] P. Colangelo and A. Khodjamirian, hep-ph/0010175.